SENSITIVITY OF NON-DESTRUCTIVE TECHNIQUE FOR DETERMINATION OF MATERIAL PROPERTIES IN REINFORCED LAMINATED COMPOSITE PLATE BASED ON VIBRATION RESPONSE

Andrejs Kovalovs, Sandris Rucevskis, Karlis Dzelzitis, Andris Chate
Riga Technical University, Latvia
andrejs.kovalovs@rtu.lv, sandris.rucevskis@rtu.lv , karlis.dzelzitis@rtu.lv, andris.cate@rtu.lv

Abstract. The paper presents a sensitivity analysis of the non-destructive numerical-experimental method for the identification of the elastic properties in laminated composite plate. The identification was performed on a laminated carbon fibre composite plate with laminate stacking sequence 0º/90º. The method is based on comparing natural frequencies between physical experiment and numerical results obtained by using response surface. Minimizing the error functional between the numerical and experimental data of vibration responses, the identification of materials properties can be performed. When solving the identification problem, it is recommended to take all experimentally obtained frequencies into the error functional. At the same time, different amounts of natural frequencies used for identification in the error functional show various values of material properties. Depending on the amount of frequencies in the error functional, different values of the elastic properties can be obtained. Eleven combinations of frequencies were used for identification of elastic properties. The percentage difference between experimentally measured and numerically calculated frequencies with various values of obtained elastic constants do not exceed 2.4%. Despite different values of elastic constants, small percentage difference shows good agreement between experimentally measured and numerically calculated frequencies. It can be seen that elastic constants obtained for the composite plate are fairly accurate in each case and can be used for modelling composite structures.

Keywords: carbon fiber-reinforced composites, frequency, response surface, elastic constants, simulation.

Introduction

The modern laminated composite materials are widely used as structural elements in civil, aeronautical, marine and mechanical engineering due to their high specific strength and design performance [1-3]. However, determination the elastic constants of laminated composite materials is much more complicated in comparison with isotropic materials and requires more effort and time. Common methods of material properties characterisation include fracture and non-destructive methods. Most popular standards include fracture methods such as compression, flexural and tensile test [4-5].

However, some materials used in testing can be expensive during manufacture that increase a cost of experiments. Non-destructive methods for determination of elastic properties can be regarded as an alternative of the fracture methods and allow to decrease the cost of experiments. In addition, the possibility to measure the elastic modulus of the specimens after processes acting on the material (temperature variations, aging, vapours, etc.) allows to reduce the consumption of material. Common methods of material properties characterisation include ultrasonic and different non-destructive tests.

The ultrasonic technique is very popular for studying of isotropic and composite materials showing good correlation between tested and reported material properties [6-7]. The well known non-destructive standard test methods for determining the material properties include the impulse excitation and sonic resonant method. Both methods show equivalency of the measured results which values do not differ significantly [8]. The non-destructive acoustic test based on longitudinal free vibration shows good agreement with fracture mechanical methods and good repeatability [9].

Vibration test is widely used for testing and understanding dynamic behaviour of structures. The change of modal parameters (mode shape, frequencies, and damping) can be used for fault identification and to verify a finite element model with good accuracy [10-11]. An inverse technique based on vibration tests allows to determine isotropic, orthotropic and viscoelastic material properties of laminated composite [12-13]. This method is based on combination of numerical-experimental models and optimisation technique with application of the error functional. The error functional includes the difference between the response of the experimental and numerical model, which has to be minimized. The influence of model [14-15] and experimental [15-16] errors on identified material properties was studied to improve the accuracy and effectiveness of the inverse technique based on vibration tests. In comparison with destructive testing of laminated composite materials, vibration testing is inexpensive and rapid.

DOI: 10.22616/ERDev.2022.21.TF193

When solving the identification problem, it is recommended to take all experimentally obtained frequencies into the error functional. At the same time, different amounts of natural frequencies used for identification in the error functional show various values of material properties. The objective of this study is identification of carbon fibre composite material properties depending on the number of frequencies used in the error functional.

Materials and Methods

In the present experiment and research, the prepreg material was used. Firstly, prepreg was cut to corresponding sizes and then layered by hand layup method. Then, a vacuum bag system was used to create mechanical pressure on the preform of the laminated composite plate during its cure cycle. The dimensions of the composite plate are 490x240 mm. The plate consists of 20 single layers with the layup 0/90 and total thickness of 1.854 mm. The density of the plate, measured by hydraulic weighting, is ρ = 1545.8 kg·m^{-3}.

The numerical-experimental method used in the present investigation consists of the several stages: (a) selection of the supposed values of elastic constants and creating the plan of the experiment; (b) numerical modelling of the laminated composite plate for the elastic constants selected; (c) approximation of the obtained data; (d) experimental testing of the laminated composite plate; (e) using the error functional for identification of elastic constants (Fig. 1).

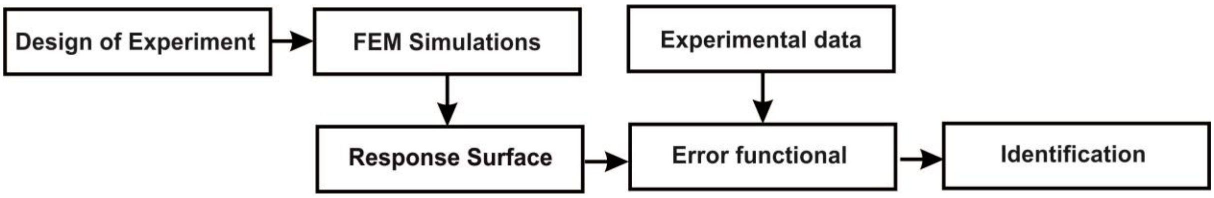


Fig. 1. **Identification method, adapted from** [17]

The Latin Hypercube design (LH) was used for elaboration of the plan of the experiment and identification of the material properties of the laminated composite plate [18]. The design of the experiment is formulated for 5 parameters of identification and 101 experiments. According to LH design, the points of experiments are distributed as regular as possible. The maximum and minimum bounds for parameters of identification are listed in Table 1. According to reference [19] Poisson's ratio ν_{23} is less sensitive to the frequencies, so it was fixed and equal to 0.325.

Table 1

Design Space

Parameters of identification	Minimum value	Maximum value
Young's modulus E_1, GPa	120.0	140.0
Young's moduli $E_2 = E_3$, GPa	7.0	10.0
Shear moduli $G_{12} = G_{13}$, GPa	1.0	5.0
Shear modulus G_{23}, GPa	1.0	5.0
Poisson's ratios $\nu_{12} = \nu_{13}$	0.2	0.4

For numerical modelling and modal analysis of the laminated composite plate, ANSYS16.0 software was used. The finite element (FE) model of the plate was built by 4-node structural shell element SHELL181. Vibration of the plate was simulated with the free-free boundary conditions. Block Lanczos method was used for natural frequencies extraction and determination of the corresponding mode shapes. Before numerical analysis, mesh convergence studies were performed to obtain the results with an acceptable accuracy.

The numerical data obtained by the finite element calculations according to the plan of the experiments was used to build the approximating functions. Elastic constant values of the laminated composite plate were approximated by a second order polynomial regression equation as shown in Eq. (1).

$$F(x) = b_0 + \sum_{i=1}^{m} b_i x_i + \sum_{i=1}^{m}\sum_{j=i}^{m} b_{ij} x_i^2 + \sum_{i=1}^{m}\sum_{j=i}^{m} b_{ij} x_i x_j \ , \tag{1}$$

where $F(x)$ – response;
x_i and x_j – values of parameters;
b_0 – constant;
b_i, b_j, and b_{ij} – regression coefficients, respectively;
m – number of the parameters.

In parallel, the laminated composite plate is tested in order to determine the vibration characteristics: natural frequencies and mode shapes, respectively (Fig. 2). The vibration characteristics were defined by POLYTEC PSV-400-B Scanning Laser Vibrometer. Hanging the plate on thin threads imitated the free-free boundary conditions. The composite plate was excited by a load speaker. The natural frequencies were obtained with accuracy of ± 0.05Hz.

Fig. 2. **Experimental set-up**

Finally, the values of elastic constants are identified using minimization of the error functional between the experimental f_i^{exp} and numerically calculated frequencies f_i^{FEM} in the relative difference:

$$\Phi_i(\boldsymbol{x}) = \sum_{i=1}^{m} \frac{\left(f_i^{\exp} - f_i^{FEM}\right)^2}{\left(f_i^{\exp}\right)^2} \Rightarrow \min . \tag{2}$$

Results and Discussion

The numerical values of the first eleven frequencies obtained by finite element modelling in the points of design experiments were used to build the approximating functions by using the program EdaOpt [20]. Frequencies have been analysed and it is found that average adjusted R^2 values are in the range 0.988–0.999. It means that the statistical models can explain more than 98.8% variability in the response and show good correlation between the results.

In the next step, the elastic constants of the laminated composite plate were obtained by minimizing the error functional from Eq. (2). The values of the first four experimental frequencies were taken to determine the elastic constants. When the first values of the elastic constants are obtained, then the fifth frequency is added. Thus, all eleven frequencies are used for identification of the elastic properties (Eq. 3). The values presented in Eq. (3) are experimentally obtained natural frequencies.

$$\begin{aligned}\Phi_i(\boldsymbol{x}) = &\frac{\left(27.1 - f_i^{FEM}\right)^2}{(27.1)^2} + \frac{\left(57.8 - f_i^{FEM}\right)^2}{(57.8)^2} + \frac{\left(82.0 - f_i^{FEM}\right)^2}{(82.0)^2} + \frac{\left(157.4 - f_i^{FEM}\right)^2}{(157.4)^2} \\ &+ \frac{\left(179.7 - f_i^{FEM}\right)^2}{(179.7)^2} + \frac{\left(209.4 - f_i^{FEM}\right)^2}{(209.4)^2} + \frac{\left(215.2 - f_i^{FEM}\right)^2}{(215.2)^2} + \frac{\left(249.6 - f_i^{FEM}\right)^2}{(249.6)^2} \\ &+ \frac{\left(307.4 - f_i^{FEM}\right)^2}{(307.4)^2} + \frac{\left(320.3 - f_i^{FEM}\right)^2}{(320.3)^2} + \frac{\left(328.1 - f_i^{FEM}\right)^2}{(328.1)^2}\end{aligned} \tag{3}$$

The identification results obtained for the composite plate with different numbers of frequencies are presented in Table 2. It can be seen that the values of elastic constants after adding of the ninth frequency are situated on the minimum or maximum bounds of the design space. Shear modulus G_{23} is equal 1.00 GPa when the total number of frequencies is 9. When ten frequencies were used in the error functional, the value of the shear modulus G_{23} becomes 5.00 GPa and the Young modulus E_1 increases until 140 GPa which corresponds to the maximum bounds of the design space. Fig. 3 presents the value of the error functional with different amounts of frequencies. It is observed that the value of the error functional significantly increases after adding the eighth frequency.

Table 2

Elastic cconstants obtained by identification

Elastic constant	Number of frequencies							
	1-4	**1-5**	**1-6**	**1-7**	**1-8**	**1-9**	**1-10**	**1-11**
E_1, GPa	130.57	130.58	131.11	131.47	130.93	136.34	140.00	140.00
$E_2 = E_3$, GPa	8.61	8.62	8.12	7.63	8.33	8.65	9.68	9.87
$G_{12} = G_{13}$, GPa	3.91	3.91	3.93	3.94	3.95	4.11	4.18	4.19
G_{23}, GPa	2.92	2.93	2.95	2.96	3.03	1.00	5.00	4.82
$\nu_{12} = \nu_{13}$	0.314	0.315	0.309	0.301	0.323	0.323	0.361	0.363

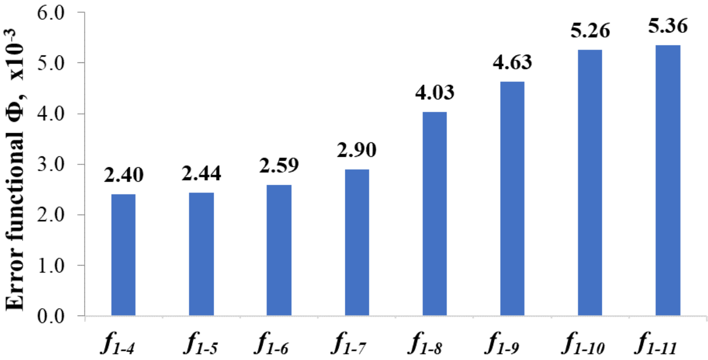


Fig. 3. **Minimum value of the error functional**

The elastic constants determined with approximations and the error functional are verified by comparing the frequencies and mode shapes numerically calculated and experimentally measured. Percentage difference of frequencies is calculated by the following expression:

$$\Delta_i = \frac{\left| f_i^{FEM} - f_i^{\exp} \right|}{f_i^{\exp}} \times 100 \,. \tag{4}$$

The percentage difference of numerically calculated and experimentally measured frequencies from Eq. (4) are presented in Table 3. It can be seen that these differences are very small and mostly do not exceed 2.4%. Despite the different values of elastic constants, small percentage difference shows good agreement between the experimentally measured and numerically calculated frequencies. It can be seen that the elastic constants obtained for the composite plate are fairly accurate in each case and can be used for modelling composite structures.

Table 3

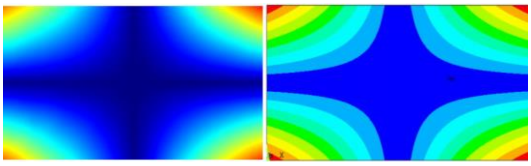
Percentage change for carbon fibre composite plate

Mode	Number of frequencies							
	1-4	**1-5**	**1-6**	**1-7**	**1-8**	**1-9**	**1-10**	**1-11**
1	1.17	1.16	1.10	1.19	1.30	1.20	0.61	0.60
2	0.04	0.03	0.31	0.33	0.29	0.22	0.29	0.33
3	2.01	2.01	2.19	2.16	2.08	2.12	2.38	2.40
4	1.11	1.12	0.84	0.83	0.86	0.86	0.87	0.82
5	0.06	0.06	0.29	0.30	0.23	0.29	0.35	0.37
6	0.78	0.79	0.40	0.27	0.47	0.49	0.64	0.64
7	1.32	1.33	0.94	0.81	1.04	1.04	1.16	1.19
8	1.38	1.37	1.71	1.81	1.59	1.65	1.57	1.53

Table 3 (continued)

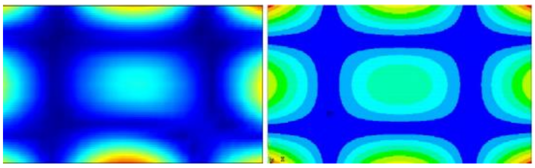
Mode	Number of frequencies							
	1-4	**1-5**	**1-6**	**1-7**	**1-8**	**1-9**	**1-10**	**1-11**
8	1.38	1.37	1.71	1.81	1.59	1.65	1.57	1.53
9	1.48	1.49	1.20	1.17	1.24	1.12	1.28	1.25
10	1.83	1.82	2.13	2.22	1.99	2.11	2.03	1.94
11	0.64	0.65	0.38	0.37	0.44	0.29	0.41	0.39

Additionally, a comparative analysis between the experimentally measured and numerically calculated mode shapes with the obtained frequencies from the identification procedure is performed. Fig. 4-5 present a four typical mode shapes for the 1st, 4th and 8th frequencies (a – experimentally measured; b – numerically calculated).



a) b)

Fig. 4. **1st frequency**



a) b)

Fig. 5. **8th frequency**

Conclusions

1. The identification procedure based on the numerical-experimental method was performed on a laminated carbon fibre composite plate with laminate stacking sequence 0º/90º. The elastic constants obtained in different combinations of frequencies were used for comparative analysis of the numerically calculated and experimentally measured frequencies.
2. The percentage differences do not exceed 2.4% for the numerically calculated and experimentally measured frequencies in vibration response with various values of elastic constants determined for various combinations of the frequencies used in the identification procedure.

Acknowledgements

This research was funded by the European Regional Development Fund Project No. 1.1.1.1/20/A/01 “A prototype of typical structural health monitoring system of operating objects for condition based maintenance”. The second author would like to acknowledge the support of the European Regional Development Fund within the Activity 1.1.1.2 “Post-doctoral Research Aid” of the Specific Aid Objective 1.1.1 “To increase the research and innovative capacity of scientific institutions of Latvia and the ability to attract external financing, investing in human resources and infrastructure” of the Operational Programme “Growth and Employment” (No.1.1.1.2/VIAA/3/19/414).

Author contributions

Conceptualization, S.R.; methodology, A.C, S.R.; software, A.K.; validation, S.R.; formal analysis, A.K, S.R.; investigation, A.K., K.D, S.R.; data curation, A.C, S.R.; writing –original draft preparation, A.K, S.R.; writing – review and editing, A.K, V.B.; visualization, A.K.; project administration, A.C.; funding acquisition, A.C, S.R. All authors have read and agreed to the published version of the manuscript.

References

[1] Hufenbach W., Kroll L., Holste C., Täger O, Barkanov E. Design of Dynamically Loaded Fiber-Reinforced Structures with Account of Their Vibro-Acoustic Behavior. Mechanics of Composite Materials, Vol. 37(2), 2001, pp. 145-152. DOI: 10.1023/A:1010673603678.

[2] Barkanov E., Eglitis E., Almeida F., Bowering M. C, Watson G. Optimal Design of Composite Lateral Wing Upper Covers. Part II: Nonlinear Buckling Analysis. Aerospace Science and Technology, Vol. 51, 2016, pp. 87-95. DOI: 10.1016/j.ast.2014.07.010.

[3] Kovalovs, A., Barkanov E., Rucevskis, Wesolowski M. Optimisation Methodology of a Full-scale Active Twist Rotor Blade. Procedia Engineering, Vol. 178, 2017, pp. 85-95.

DOI: 10.1016/j.proeng.2017.01.067.
[4] Lasenko I., Grauda D., Butkauskas D., Sanchaniya J. V., Viluma-Gudmona A, Lusis V. Testing the Physical and Mechanical Properties of Polyacrylonitrile Nanofibers Reinforced with Succinite and Silicon Dioxide Nanoparticles. Textiles, Vol. 2(1), 2022, pp. 162-173. DOI: 10.3390/textiles2010009.
[5] Bleive L. L, Lusis V. Experimental study and numerical modelling for flexural capacity of FRC structural elements. Proceedings of the 13th International Scientific and Practical Conference "Environment. Technology. Resources." June 17-18, 2021, Rezekne, Latvia. Vol. 3, pp. 154-158. DOI: 10.17770/etr2021vol3.6661.
[6] Meza C. A., Franco E. E., Ealo J. L. Implementation of the ultrasonic through-transmission technique for the elastic characterization of fiber-reinforced laminated composite. DYNA, Vol. 86(208), 2019, pp. 153-161. DOI: 10.15446/dyna.v86n208.70279.
[7] Munoz V., Perrin M., Pastor M.-L., Welemane H., Cantarel A, Karama M. Determination of the elastic properties in CFRP composites: comparison of different approaches based on tensile tests and ultrasonic characterization. Advances in Aircraft and Spacecraft Science, Vol. 2(3), 2015, pp. 249-261 DOI: https://doi.org/10.12989/aas.2015.2.3.249.
[8] Húlan T., Obert F., Ondruška J., Štubna I., Trník A. The Sonic Resonance Method and the Impulse Excitation Technique: A Comparison Study. Applied Sciences, Vol. 11(22), 9 P. DOI: 10.3390/app112210802.
[9] Mousavi S.Y., Jalili M.M., Pirayeshfar A.S. Non-Destructive Acoustic Test (NDAT) to Determine Elastic Modulus of Polymeric Composites. In proceedings 29th European Conference on Acoustic Emission Testin,g 8-10 September, 2010, Vienna, Austria, p7.
[10] Kovalovs A., Rucevskis S., Akishin P., & Kolupajevs J. Numerical Investigation on Detection of Prestress Losses in a Prestressed Concrete Slab by Modal Analysis. IOP Conference Series: Materials Science, Vol. 251(1), 2017, 6p. DOI: 10.1088/1757-899X/251/1/012090.
[11] Glukhikh S., Barkanov E., Masarati P., Morandini M., Riemenschneider J., & Wierach P. Design of helicopter rotor blades with actuators made of a piezomacrofiber composite. Mechanics of Composite Materials, Vol. 44(1), 2008, pp. 57–64. DOI: 10.1007/s11029-008-0007-9.
[12] Rikards R., Abramovich H., Green T., Auzins J., Chate A. Identification of Elastic Properties of Composite Laminates. Mechanics of Advanced Materials and Structures, Vol. 10(4), Pages 335 – 352, October 2003. DOI 10.1080/10759410306755.
[13] Barkanov E., Chate A., Rucevskis S, Skukis E. Characterisation of Composite Material Properties by an Inverse Technique. Key Engineering Materials, Vol. 345-346, 2007, pp. 1319-1322. DOI: 10.4028/www.scientific.net/KEM.345-346.1319.
[14] Wesolowski M, Barkanov E. Model Errors Influence on Identified Composite Material Properties. Composite Structures, Vol. 94, 2012, pp. 2716-2723. DOI: 10.1016/j.compstruct.2012.03.030
[15] Barkanov E., Wesolowski M., Hufenbach W, Dannemann M. An Effectiveness Improvement of the Inverse Technique Based on Vibration Tests. Computers and Structures, Vol. 146, 2015, pp. 152-162. DOI: 10.1016/j.compstruc.2014.10.006.
[16] Wesolowski M, Barkanov E. Air Damping Influence on Dynamic Parameters of Laminated Composite Plates. Measurement, Vol. 85, 2016, pp. 239-248.
[17] Kovalovs, A., & Rucevskis, S. Identification of elastic properties of composite plate. IOP Conference Series: Materials Science and Engineering, Vol. 23, 2011, 012034.
[18] Audze P, Eglais V. New approach to the design of multifactor experiments. Problems of Dynamics and Strengths, Vol. 35, 1977, pp. 104-107.
[19] Rikards R., Bledzkij A.K., Eglajs V., Chate A., Kurek K. Elaboration of optimal design models for composite materials from data of eksperiments. Mekhanika Kompozitnykh Materialov, Vol. 4, 1992, pp. 435-445. DOI: 10.1016/B978-0-444-89869-2.50016-0
[20] Auzins J., Janushevskis A., Janushevskis J, Skukis E. Software EdaOpt for experimental design, analysis and multiobjective robust optimization. Proceedings of the International Conference on Engineering and Applied Science Optimization. 4-6 June, 2014, Kos Island, Greece, pp. 101-123, 2014.